Fig. 1 *Helianthus annuus*

Spiralling into oblivion – Fibonacci & plants

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One of the challenges of waiting for one's passenger after an HPS committee meeting at Pershore College is the well-known hazard of the specialist horticultural library. On one such occasion, random snooping around the books led me to several references to the Fibonacci number series and its occurrence in the structures of some plants. The most common example, quoted widely, is the pair of spirals seen in the 'face' or disc florets of *Helianthus annuus* (fig. 1): it's easy to see one set spiralling in from the edge in a clockwise fashion, the other anti-clockwise. The books usually invite you to count the number of spirals of each set and be duly amazed that these turn out to be adjacent numbers (21 & 34) in the Fibonacci series (the series begins 1, 2, 3, 5, 8, 13, 21, 34 – we'll come back to this later).

Do the plants 'know' this series? Do they really 'know' some mathematics? And what is more, is there some hidden advantage in having such attractive features in one's face? There is no doubt that the spirals add great beauty to plant morphology as far as ***we*** are concerned, but how do the spirals arise – is there some underlying evolutionary benefit, and is it something genetic or is it simply physics?

Fig. 2 *Bellis perennis*

Fibonacci and his 'series'

The man we know as Fibonacci was more generally known in his day as Leonardo da Pisa (ca. 1170–1250). He was an outstanding mathematician who was involved in introducing algebra to Europe. Leonardo's father, Sr. Bonacci, was also famous, so Leonardo junior often signed himself 'filius Bonacci' or simply fi-Bonacci (son of Bonacci).

Fig. 3 *Leucanthemella serotina*

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Fig. 4 *Kniphofia rooperi*

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Fig. 5 *Dipsacus laciniatus*

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Fig. 6 *Echinacaea purpurea*

Fibonacci's series starts with '0', then '1', and continues with the rule (an algorithm) that the next number is always the sum of the previous two numbers; check it out with the values above. It looks harmless enough, but it has spawned hundreds of serious mathematical books and papers, a literature of imaginative tosh, and of course it has expression in the plant world. Look out for both spirals in the simple daisy, *Bellis perennis* (fig. 2), with 13 & 21 spirals, and in the less simple *Leucanthemella serotina*, (fig. 3), with 21 & 34 spirals.

Many authors on the subject refer to the 'numbers of petals' in a flower, claimed to 'always' be a Fibonacci number; 'often' would be more accurate, as 3, 5, 8, etc. are indeed common but, as we know, the crucifers with their 4 petals don't seem to have heard of Fibonacci! These examples have been drawn from the flowering plants, but we don't have far to go to find spirals in the arrangement of the scales in conifers. Peer into botany text books and there the spirals are again under the heading 'phylotaxis', which we'll come to later.

Try googling 'Fibonacci' and be amazed, and be wary! There are claims that the series is revealed in the dimensions of the famous pyramids in Egypt and in some ancient Greek temples. There are books on using the Fibonacci ratios to play the stock market. One of the strangest is an attempt to link the revolutionary periods of planets around the sun to plant phylotaxis. The field is littered with 'selective choice of evidence' and a preference for obscure explanations over simple ones.

Back to Plants

Once one is aware of spirals, they are seen in many species. For example, look at the florets in *Kniphofia rooperi* (fig. 4) with numbers 8/13; the spiky bosses of *Dipsacus laciniatus* (fig. 5) numbers 5/8; the spheroidal bosses of *Echinacaea purpurea* (fig. 6); and even the compound residues of the flowers in pineapples (fig. 7). (Note that decorative stone pineapples (fig. 8) do not sport Fibonacci spirals as they have an equal number of bumps and are quite symmetrical.)

And it is not just the flowering parts: not surprisingly, the arrangement of the seed maintains this spirally phenomenon, such as *Geum urbanum* (fig. 9). Sometimes the spirals are not easy to see, for example the leaf arrangement in *Euphorbia characias* confuses the eye (fig. 10), but look at the euphorbia leaf scales and there they are, spirals in two directions and with different slopes (fig. 11)!

Fig. 7 Pineapple

Fig. 8 Stone pineapple

Fig. 9 *Geum urbanum* seeds

This spiralling round a stem directs us to the botany books and to the term "spiral phylotaxis". Phylotaxis is the way leaves are arranged along stems; in this euphorbia the leaves seem to spiral around as we go up the stem. Taking any leaf as the starting point, we count the number of **new** leaf axils we pass until we get to a leaf axil which is **exactly** above the starting point (easier said than done). We also count the number of 'complete turns around the stem' to get there. Fig. 12 makes this clearer: we pass 8 axils and use 3 turns, so the phylotaxis is called 3:8 (again Fibonacci numbers).

The interpretation of these spirals was a challenge which left even Charles Darwin baffled. Some writers refer to them as 'Golden Spirals' or even 'spira mirabilis' (miracle spirals). But are spirals really such an unexpected phenomenon? Moving briefly away from horticulture we come to our friend the snail with the spirals in his shell (does he know maths as well?) and fossils – fig. 13 shows fossilised roots or stigmaria (found in coal measures laid down about 300 million years ago); look closely and there are the spirals again. Spirals are more common than we thought and they have been around for a very long time.

Spirals and conifers

The fossil in fig. 13 suggests that we might do well by looking into the past; for plants we can do this by looking at the family *coniferales*, a family which displays in the fossil record evidence of spirals in cones and trunks. It is relevant to recall that conifers were a dominant family millions of years **before** our beloved flowering plants; extant conifers such as the Korean fir, *Abies koreana* (fig. 14), and the noble fir, *A. procera* (fig. 15), display spiralling in the vast majority of their cones. Usually one spiral is easy to see and on close inspection there is another at a different pitch and going the other way round. And yes, the numbers of spirals in each direction are again adjacent Fibonacci numbers, in this case 5:8 for *A. procera* and 8:13 for *A. koreana*. However, although the conifer spirals are often thought similar to the spirals on the sunflower disc, there is an important difference: even though the basal part looks confused, it is clear that in cones the number of each spiral remains the same as growth progresses, but the spirals are more like distorted helices. The needleless spikes in fig. 16 show that the helical spirals originate at the central spike of the cone and, further, that at least in conifers the direction of spirals of the same pitch can be different on two cones from the same plant. Look closely at fig. 14 to see this here as well.

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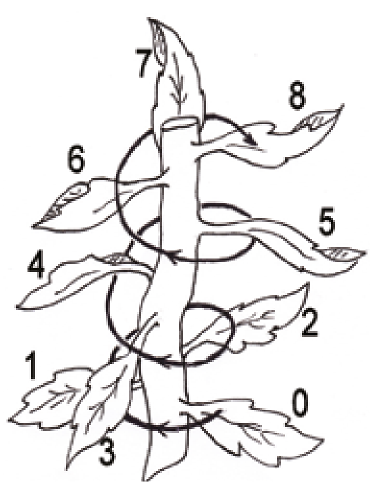

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Figs 10, 11 & 12 *Euphorbia*

We can go further with conifers by looking at the Monkey Puzzle tree, *Araucaria*, (fig. 17). This shows the spiralling of sharp leaflets round the trunk and the numbers of each are still adjacent Fibonacci numbers, 13:21 (sometimes 15:24!), but closer inspection reveals a third spiral. The spiralling in conifers clearly predates our relatively recent flowering plants, so how do we link twirling around a trunk to the display in sunflowers and asters? We start by looking down the stem of a growing tip of the araucaria (fig. 18). A few painful hours with gloves and pliers applied to stripping away the leaflets in reverse order of growth enabled me to number each leaflet, the lowest number for the youngest leaf. The starting point is arbitrary as the smaller leaflets are too small to deal with, but what is revealed gives an approximate fit to theoretical expectations that the leaflets in the same sector will have the Fibonacci relationship: so starting from leaflet marked 0, we get numbers 3, 5, 8, 13 and 21 radiating outwards. An arbitrary start at say bud 3 radiates out to values of '3 plus the Fibonacci numbers' i.e. 6, 8, 11 and 16.

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Fig. 13 Fossilised roots or stigmaria

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Fig. 14 *Abies koreana*

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Fig. 15 *Abies procera*

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Fig. 16 Helical spirals originate at the central spike of the cone

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Fig. 17 Monkey Puzzle tree, *Araucaria*

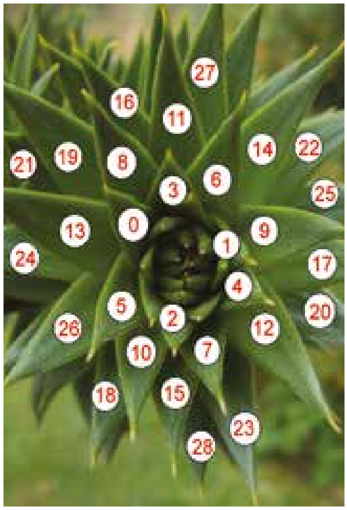

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Fig. 18 A growing tip of *Araucaria*

The text books on the subject seem to get a better fit but perhaps my tree has not read the books!

What plants 'do not want'

We'll look at how well sunflowers and company behave in the light of theory, but first we need to ask what the plant does **not** want! All plants construct the vast majority of their constituent macro-molecules from the extremely small molecule carbon dioxide via long-winded biochemistry, a consequence of which is evolutionary selection favouring high efficiency in all things. Thus plants 'do not like' waste of space or resources. Buds or seed which 'sit in the nick' are more efficient than square arrangements, as are space-filling spirals rather than space-wasting cart-wheel spokes.

Fibonacci and ϕ

Ancient Greek mathematicians were very interested in ratios. We all know about their discovery of the ratio π for circles and spheres; they also discovered the ratio ϕ (pronounced "fie"). ϕ is the number obtained by dividing any larger Fibonacci number by its preceding partner[1] and has the value 1.61803... and like the decimal expression of π, it never comes to the end of the decimals.

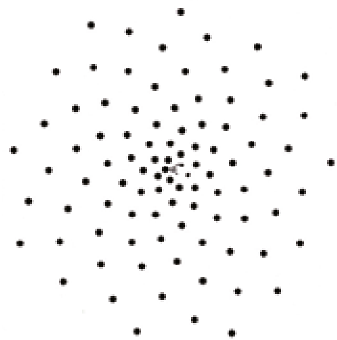
ϕ crops up all over geometry in most surprising places. We're going to use it in a simple geometrical model and see how the result compares with our actual observations of plants. The central question is why are there Fibonacci spirals?

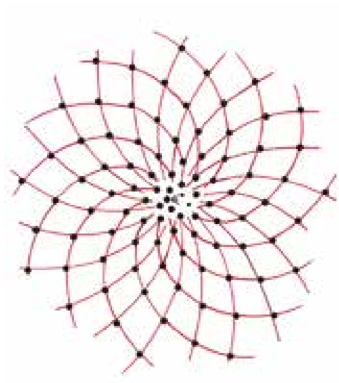
The model system using phi

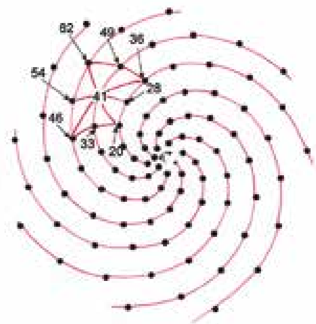
We start with the primordia or proto-buds being produced sequentially by the meristems and characterised by the rule 'grow a bit and turn a bit'. Here the 'turn a bit' is the decimal part of ϕ as a fraction of a circle[2] (that is about 138°, the 'golden angle'). As more buds are produced, the

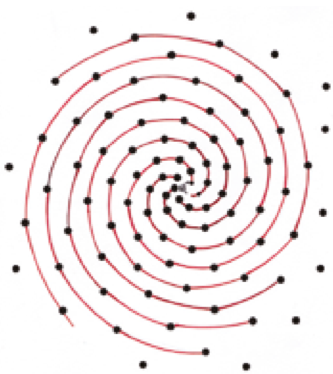
[1] The ancient Greeks reasoned as follows. Take a stick of length c; can we break it into two parts such that the ratio of the original stick to the larger broken piece b is equal to the ratio of the piece b to the broken piece a? Remembering that c is actually a-plus-b, we write this in algebra as: $(a+b)/b = b/a$. The value for which this is true is 1.61803, or ϕ. Try writing out a Fibonacci sequence starting with a & b!

[2] That is 360° multiplied by 0.61803 to get 222.4908 ° (we ignore the 1 because this is just one full circle.) Remember that this is the same thing as 137.509° but turning the other way! About 138° works well enough for our plants.









Figs 19, 20, 21 & 22 'The golden angle' (about 138°) – the challenge is our perception

previous buds simply move outwards along a radius as the primordia develop. The value of the golden angle means that each successive bud is located about a third of a circle **away** from its predecessor. The first 90 or so buds are shown in fig. 19. At first glance there is not much order (I have slightly lengthened the radial part to make it clearer). There is a clue when we notice that for the buds nearer to the edge, the spirals can be made out (as we find in the plants themselves).

Drawing in the spirals reveals one set of 21, and another of 13 members (fig. 20).

The lower numbered spirals are still there, we just don't see them as easily. I have marked them in figs 21 & 22, still Fibonacci numbers, 8 & 5. The challenge here is **the way we see** spatial relationships! We are concerned with near neighbours in the disc florets, not near in the sense of near-bud-numbers. Some neighbours are almost shoulder to shoulder, others are almost (but not quite) vertically aligned. You can see that the higher the number of spirals the more vertical the spiral lines are. In fig. 21 I have chosen a random bud, in this case bud number 41, shown in the upper left section along with its eight closest neighbours and their bud numbers. Remember that these are the eight neighbours as we see them, look closely at the differences in the bud numbers between no. 41 and the others, and what we get are our old friends the Fibonacci numbers. And also take note that all of the spirals go through no. 41, not just the two most obvious ones. The spirals arise as a direct consequence of the properties of ϕ (which we used to provide the angle at which primordia are produced) **and** the way our brains see things. Another way of looking at this, (which boils down to the same thing) is that the set of say 5 spirals is simply the result of joining up every fifth bud number; the set of 8 spirals is the result of joining up every eighth bud number; and so on.

Theory and reality

Horticulture is a field in which practice and observation trump theory. There are many cases in which observation does not fit Fibonacci numbers, or perhaps we should say there are many cases for which the Fibonacci ideas don't fit reality. It does not detract from the Fibonacci interpretation to find that whole families, for example the crucifers, seem oblivious of this system of magnificent mathematics. Nor do the casual observations of non-compliance on the same plant challenge this; for example, teasels from our own borders also show 26:31 and 26:41 as well as the 'predicted' values 21:34. Playing around with the computer model shows that even changing the golden angle a bit doesn't necessarily lead to the collapse of the spirals. As usual, botany seems to be content with being approximately consistent!

So we conclude that evolution has selected for a turn-a-bit angle of about 138° leading to optimisation of resources and a geometrically efficient way of dealing with curved surfaces. The famous spirals we see are a spin off from the use of ϕ; the spirals in themselves are unimportant to the plants, although to us they remain things of beauty.

Is it genes or is it physics?

A fair question to ask is whether the spirals (whether they be true logarithmic spirals, distorted helices, or whatever) are the result of purely physical forces, and the efficient use of space and materials. I tried jiggling 200 pennies around on a flat surface, resulting in a few curvy lines but nothing spiral like the plants. Perhaps a better model is jiggling peas around within a curved surface like a mixing bowl: several spirals appeared but not enough to claim the cause is physics alone.

So how do the plants do it? Current botanical conjecture goes something like this. The growing tip of the stem has a multi-cellular tissue called the meristem; cells are dividing rapidly to produce buds called primordia located just behind the advancing tip (shown schematically in fig. 23). There is thought to be a time interval (called a plastochrone) between the production of successive primordia, and space constraints and genetic features together lead to the expression of successive primordia at about the golden angle of 138°– a sort of pulsating production of the primordia. And how does the plastochrone operate? Again we are reliant on conjecture, that

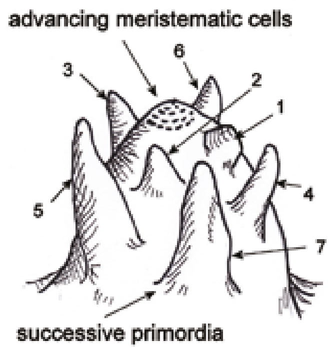


Fig. 23 Cells in the meristem divide rapidly to produce buds (primordia) behind the advancing tip

is we believe that the genes can 'switch on 'and 'switch off' the production of both inhibitor and promoter hormones which are thought to have different diffusion rates within the cellular fluids which produces the pulses. This is not as far-fetched as it might sound; there are some pulsating/oscillating systems of this sort with totally inorganic components (known as Belousov-Zhabotinsky reactions). Carried into cell biology we are back in the land of grow-a-bit-turn-a-bit.

So are Fibonacci's spirals the result of genes or of physics? It seems to be a bit of both.

Derek Cooper read chemistry at UMIST followed by six years research at the universities of Manchester, Cornell and Cambridge and teaching at Staffordshire University. His interest in horticulture came on the coat-tails of his wife Pauline's wider & deeper knowledge, but the scientific habit of a lifetime – asking 'why and how?' – always surfaces sooner or later.